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Depth dependence of vertical plunging force in granular medium

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Depth dependence of vertical plunging force in granular medium is studied experimentally by measuring the slow-pushing force of different size and shape objects intruding vertically into a granular bed. It is found that all of the force curves of fully immersed intruders have concave-to-convex transition. The depth dependence of the force turns from supralinear to sublinear. By studying the properties of the inflection point of the concave-convex transition, we find that the plunging force at inflection point is proportional to intruder's volume, and the inflection point occurs when the intruder is fully buried to a level around twice its diameter. Testing by plunging a long cylinder, which is always partially immersed, we find no inflection point in this case, which verifies that the inflection of the plunging force is related to the filled-in loose granules on top of the intruder. The slowdown of the increasing rate of the force is, therefore, not a result of sidewall support proposed by previous researchers.

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I. INTRODUCTION

Although being processed frequently in industry and engineering, granular matter is still poorly understood in its dynamical and static behavior [1,2]. A fundamental problem such as stress distribution or force propagation in static granular medium has not been solved completely. Except for some special cases [3], there is no universally accepted theory for stress calculation yet [4]. Most peculiar behaviors of static granular medium, which differ from that of general solid or liquid, could attribute to the characteristic fractal distributed stress structure, namely, force chains. Properties of force chains, e.g., distribution and reorganization of force chains, are emphasized in the recent study of the static or jammed granular matter [5–11]. Through measurement of the response function of a granular system with a local point force on the top, the physical image of force propagation in quasistatic granular medium is established, which states that the propagation form is wavelike, where the force is propagated along some characteristic directions in short range, and elasticlike in larger dimension [9]. A numerical simulation shows that the elastic regime can be enlarged with increasing disorder and friction of the system [10,11].

The force propagation problem considered above is about force chain distribution with no change in the interior configuration of the medium in the response function measurement. In industrial processes problems are always more complicated; the jammed granular matter in such processes would usually be under plastic deformation or with slow flow, and change its interior configuration, i.e., the force chains would be reorganized.

Depending on a dimensionless shear rate, the flow of granular matter can be divided into rapid, intermediate, and slow flow regimes [12]. In slow flow regime the major interaction between individual particles is friction, and thus it behaves differently from liquids. A fluid mechanistic approach has been proposed for slow frictional flow of powders, and comparisons of powder and fluid flow in identical geometries are presented [13]. An object immersing and moving slowly in granular medium provides a way to detect the dynamics of force chains structure, which is a problem important in many applications.

Albert *et al.* [14] studied the drag force of an inserted cylinder moving slowly and horizontally in granular medium, and found that the drag force was linearly dependent on both the depth of the insertion and projected area of the cylinder. Another horizontal experiment shows that, if the object is fully immersed into granular bed, the depth dependence is not linear but supralinear [15]. The vertical drag force of an intruder moving in granular medium has also been measured. Stone *et al.* pushed a flat plate vertically into a granular medium [16,17], and found that penetration force increasing with the penetration depth was nearly linear in initial regime, then followed by a depth-independent regime. When the plate was pushed near the bottom of the container, the penetration force showed an exponential increase. The authors deem that the initial linear regime is due to hydrostatics while the depth-independent regime is a Janssen-like regime, which is due to sidewall support. Hill *et al.* [18] measured the drag force of an intruder plunging into and withdrawing from a shallow granular bed (about 100 mm), and found that both the plunging and the withdrawing forces had power-law dependence on the immersion depth with exponents greater than unity, i.e., 1.3 for plunging and 1.8 for withdrawing. In our previous work, when studying an object sinking in a granular bed by its own weight, a power-law relation between the supporting force of granular bed and sinking depth with exponent greater than one was observed [19].

In this work, we report a slow plunging experiment done by pushing vertically different shape intruders into granular bed deeply (yet not too deep to sense the effect of the bottom of the container). It is found that the plunging force is increased rapidly with immersion depth in shallow regime, and the increasing rate slows down in deeper regime, similar to the force curve obtained in Ref. [17]. In the shallow regime plunging force depends supralinearly on depth, different from that of hydrostatic case, while in deeper regime, the dependence changes to sublinear. More importantly, the sublinear dependence is found not as a result of the wall support as was claimed [17], but is a result of volume effect due to the passage of the intruder in the granular medium.

II. EXPERIMENTAL SETUP AND METHOD

In the experiment, a cylindrical steel container with diameter of 19.6 cm and height of 25 cm filled with monodisperse glass beads (density 2.46 $\mathrm{g/cm^3}$) with diameters of 0.4–0.5 mm is used. Intruders of different shapes and sizes are attached via a thin rod (4 mm in diameter) to a force transducer (resolution of 0.1 g), and the force transducer is fixed to a motorized vertical translation stage. The translation stage is controlled to push the intruder into the granular bed vertically at a constant rate of 0.5 mm/s. Three shapes of intruders are used in the experiment: spheres, cylinders, and cones. The radial length D and axial length h of the intruders are chosen to be the same, i.e., $D=h$ for all cylinders and cones, so that we can use one parameter (size D) to denote them. Nine cylinders and nine cones with D ranging from 10 to 30 mm and eight spheres with diameters D ranging from 10 to 35 mm are used. The cylinders are vertically oriented, and the cones are pointing down during the experiment. Surface of all objects are smooth to minimize the shear friction.

The resistance force that the granular bed imposed on the intruder during the process is recorded by the force transducer. The bed is prepared with similar procedure as method 1 of Ref. [20] before each measurement. To ensure the same initial condition the same steps are repeated before each measurement: the glass beads are poured through a hopper placed 5 cm above the container until the whole container is filled, and the surface of the bed is leveled flat after each fill. The volume fraction of the prepared bed is $60\% \pm 0.5\%$. Preparation method has been tested and compared with a different procedure with hopper position kept slightly above the apex of bed. No significant difference is found in the force curve $f(z)$ between these two procedures. We, therefore, choose the simpler preparation method.

Humidity during the experiment is 10–30 %; in this humidity range the adhesion is minor and experiment results are reproducible. The intruders are pushed into the granular bed for a depth of at least 150 mm. The size of the glass beads is much smaller than the intruder size, and the difference of the plunging force curves of the same intruder in different runs is minor. Therefore, just three runs of each intruder are recorded for average.

III. EXPERIMENTAL RESULTS

Figure 1 shows the depth dependence of the plunging force experienced by eight spheres when being pushed slowly into a granular bed. As shown in Fig. 1, all of the plunging forces show immersion depth dependence having a transition from supralinear to sublinear.

Similar curves were obtained by Stone and co-workers when measuring the penetration resistance of a plate pushed into granular medium [16,17]. They explain that the initial region is roughly linear, owing to hydrostatic pressure, while the following nearly flat region is due to Janssen effect, where the sidewalls of the container support a significant portion of the weight of the grains. If their claim is correct, then in a container that is wide enough to ignore the coupling effect between intruder and sidewalls, the penetration resistance of different size intruders shall become depth independent at nearly the same depth because the medium pressure in the container is unrelated to intruder's size, just a function of depth and parameters of the granular system. However, as shown in Fig. 1, although all the force curves turn from supralinear to sublinear dependence of depth after pushing to a certain depth, the inflection points are different for different sphere diameters. Moreover, as known in Janssen effect, the wall support would be taken into account until the silo is filled to a height about two times of the silo diameter. While in our experiment (same as Stone's experiment), the diameter of the container is comparable to the height and, therefore, the side-wall effect should be minor. This phenomenon suggests that the sublinear depth dependence of the resistance should not be the result of the support of sidewalls, or say Janssen effect. In fact, all the curves in Refs. [16,17] also have similar concave-to-convex tendency at a closer inspection.

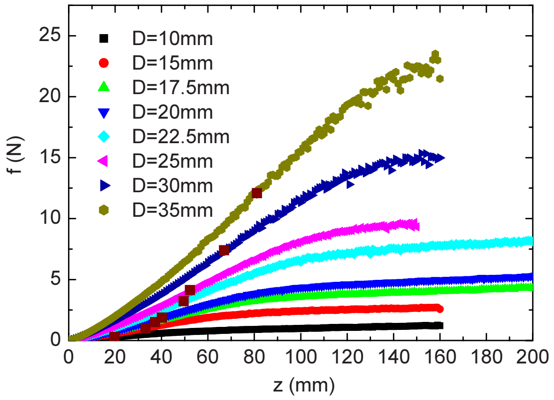

FIG. 1. (Color online) Plunging force f of sphere intruders with different diameters D as a function of penetration depth z. The inflection point is indicated by a large brown (dark) solid square on each curve.

The smoothed curves $f(z)$ were differentiated in first and second orders. The differential result confirmed the existence of the transition from concave to convex of the force curves, and the inflection points are shown in Fig. 1 by the square brown points. At the inflection point z_0, $f''(z_0)=0$ and $f''(z_0)$ changes sign from positive to negative, which means that the change rate of $f(z)$ turns from accelerating to decelerating. The depths z_0 increase with diameter of sphere D, as is manifested in Fig. 1.

Figure 2 shows the diameter dependences of depths z_0 and plunging forces $f(z_0)$. As shown in the inset of the figure, z_0 is the linear dependent of D, with a slope of 2.4, and $f(z_0)$ is linearly dependent on D^3 or the volume V of the sphere. These suggest that the emergence of inflection point or the slowdown in growth of plunging force is related to a volume effect of the intruder in granular medium.

Since the plunging force at the inflection point is proportional to the volume of the intruder, it should be expected that the plunging force be scaled by the volume of the in-

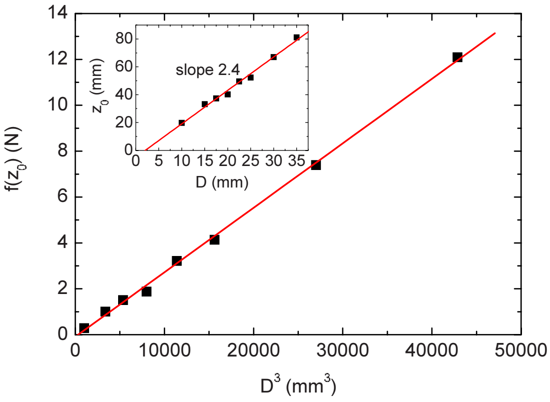

FIG. 2. (Color online) Plunging force f (z_0) at the inflection point of each curve in Fig. 1 is plotted as a function of D^3. The solid line is a linear fit. The inset is a plot of z_0 as a function of D.

truder. When the immersion depth z is rescaled by intruder diameter D, and the force f is rescaled by D^3, the data in Fig. 1 collapse within our data range, as is shown in Fig. 3. It can be seen that the data collapse well in shallow regime before the inflection point emerges but not as good as in deep regime. In the deep regime, the rescaled forces for the smallest intruders are greater than those for the larger intruders. This imperfect collapse may be related to the small ratio of D/d for smaller intruders, where D and d are diameters of the intruders and the glass beads, respectively. In Ref. [21], Soller *et al.* find that the small ratio of D/d may cause the increase in the drag force of a rotating vane in the granular bed. By taking into consideration the dimension of glass beads, and using effective depth and intruder diameters, the force curves for different sizes of beads can collapse well [21]. However, as the same modification was applied to our data, the improvement of the collapse is limited. It should be noted that the smallest ratio of D/d in Ref. [21] is almost one while in our experiment the smallest ratio is greater than 20. Therefore, the little improvement of the collapse shall be expected. The failure of the rescaling in deep regime suggests that there are still some essential differences between granular medium and continuous liquid, and need more investigation. The log-log plot in Fig. 3 suggests different depth dependences of plunging force in different regimes. Within shallow regime, i.e., before reaching the inflection point, the plunging force fits to power-law dependent for all spherical intruders, with exponent about 1.3 ± 0.02. It should be noted that in Ref. [18], exponent of 1.3 was also obtained in depth dependence of their plunging force. While in the deep regime, the depth dependence becomes sublinear.

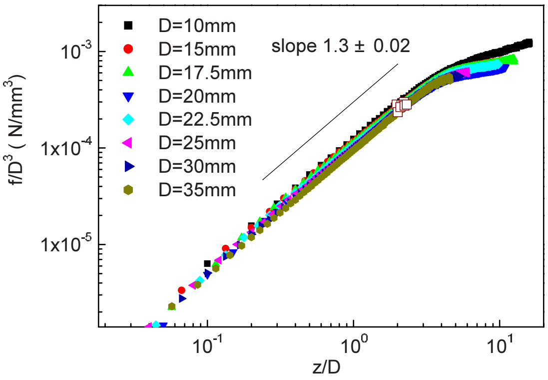

FIG. 3. (Color online) Rescaled plunging force f/D^3 as function of rescaled depth z/D in log-log plot for sphere intruders. The solid line with slope of 1.3 is a fitting of data points before the inflection points. The open points in the figure indicate the inflection point.

The concave-to-convex transition of plunging force curve $f(z)$ and the essence of volume effect of the inflection point are not peculiar properties of spherical intruder. Cylindrical and conical intruders show similar phenomena. Figures 4(a) and 4(b) are depth dependence of plunging forces of cylindrical and conical intruders, respectively. The inflection points are shown in the figures. For these two shape intruders, $f(z_0)$ also show linear dependence on the volume V of the intruder, and z_0 depends linearly on D but with slope of 3.7 for conical intruders and 2.8 for cylindrical intruders (not shown in the figures).

For cylindrical and conical intruders, the log-log plot of rescaling force also shows supralinear depth dependence in shallow regime and sublinear depth dependence in deep regime, as shown in Figs. 4(c) and 4(d). The lines with slope of 1.3 are drawn to guide the eye.

IV. DISCUSSION

When an object is slowly vertically pushed into a Newtonian incompressible fluid, where the velocity-dependent viscous force is negligible comparing to buoyancy, the drag force experienced by the object should eventually be nearly equal to the buoyancy, which is proportional to the exclusive volume of the object and is independent of penetration depth. The volume-effect buoyancy can be considered as the difference of the pressure acting on the top and the bottom of the object. It can also be used to explain the change in z dependence of the plunging force in the granular medium. When a sphere is pushed into the granular bed vertically, the granules at the moving front of the sphere will be locally fluidized and pushed laterally first, and then avalanche follows as the sphere passes through, which fills the passage left behind the intruder, just like fluid filling in a cavity. The filling brings pressure on top of the intruder and counteracts a part of the pressure from the bottom and, therefore, reduces the plunging force increasing rate. Recalling that, in silo effect the wall support would be taken into account until the silo is filled to a height of about two times of the silo diameter, it is also reasonable to have a linear relation between the depth of inflection point, z_0, and the intruder's lateral dimension D shown in the inset of Fig. 2. Therefore, the emergence of inflection point is a signal that the filled-in granules on top of the intruder have been reorganized in bulk, and this fill-in effect shall be taken into account in the plunging force.

To test the validation of the above volume-effect explanation for the change from supralinear to sublinear dependence of the resistance, we performed an experiment by pushing a long rod with length of 230 mm and diameter of 4 mm into a granular bed of 220 mm in depth. During the whole process, the rod was never fully immersed. Therefore, one should expect no transition from concave to convex. Indeed

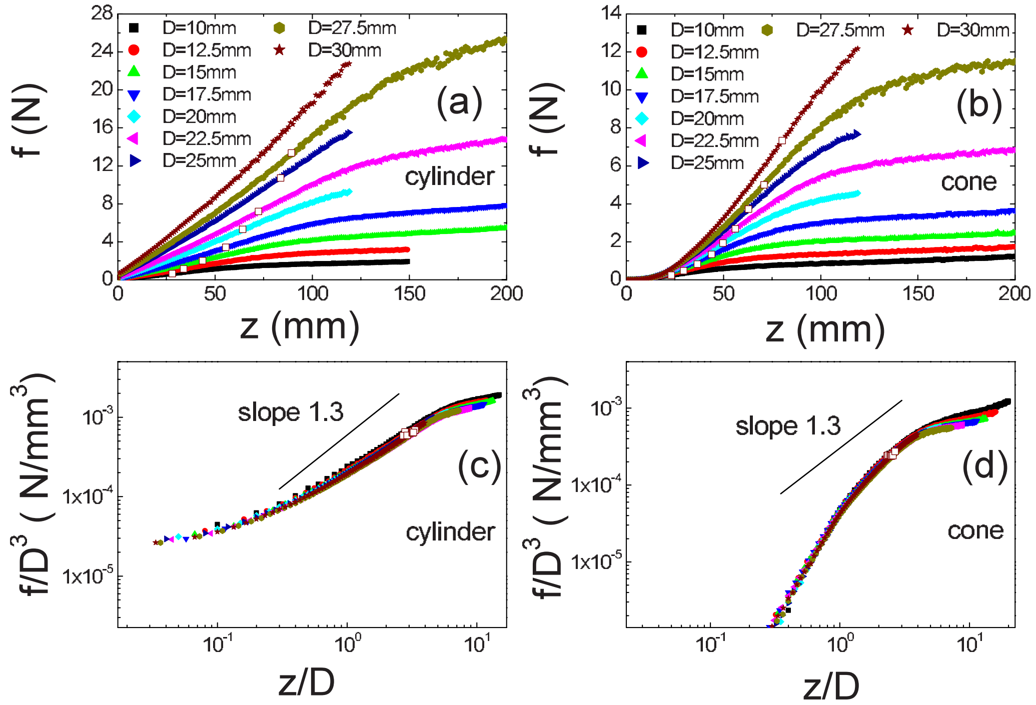

FIG. 4. (Color online) Plunging force f of cylinder and cone intruders is plotted as a function of z. Since each cylinder and cone intruder has the same width and height, we use only the parameter D for their sizes. Plunging $f(z)$ are plotted for different D for (a) cylinder and (b) cone. Replotted in (c) and (d) of the rescaled plunging force f/D^3 versus depth z/D in log-log plot for (c) cylinders and (d) cones. The solid lines with slope of 1.3 is drawn to guide the eye. The open points in all the curves are the inflection points.

as is shown in Fig. 5, the plunging force shows a power-law dependence on depth with exponent of 1.2 in the whole region, which is close to exponent of 1.3, similar to the result of Hill's plunging experiment [18].

In Ref. [18], Hill *et al.* has concluded a model to estimate the average pressure for granular bed on the advance surface of the intruder, and found that pressure P in the plunging experiment is not linearly dependent on immersion depth z but follows a supralinear form, $P \sim z^{1.3}$. In this work, our plunging experiment confirmed that the depth dependence of plunging force in shallow region is supralinear. However, the exponent is not exactly 1.3 for all shape of intruder. For spherical intruder, the exponent is almost 1.3 (shown in Fig. 3), while for cylindrical and conical shape intruders [shown in Figs. 4(c) and 4(d)], their slopes in shallow regime are different. For cylindrical intruder, the exponent is smaller than 1.3 (about 1.0–1.1 for $z/D > 1$), and for conical intruder, the exponent is greater than 1.3 (about 1.8 for $z/D > 1$). The reason that the exponents for these two nonspherical intruders differ from those of spheres is possibly due to the sharp edge of their odd shapes. In the deep regime, with the filling-in effect as mentioned above, the depth dependence of plunging force turns from supralinear to sublinear for all three shape intruders [Figs. 3, 4(c), and 4(d)]. As shown in Figs. 1, 3, and 4, the plunging force of cone is smaller than that of sphere and cylinder with the same sizes in shallow regime, which means that the cone shape is easier to plunge as shall be expected.

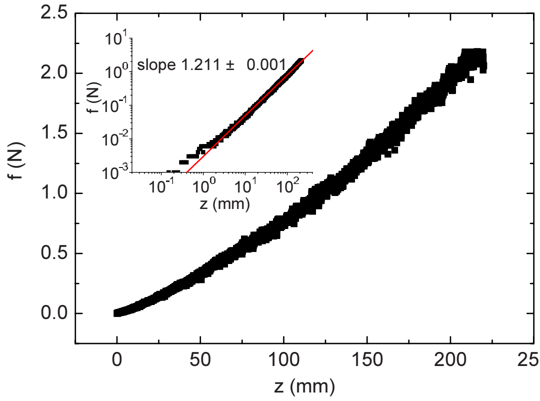

FIG. 5. (Color online) Depth dependence of plunging force f of a long cylindrical intruder (230 mm in length and 4 mm in diameter), which is never fully pushed into granular bed. The inset log-log plot shows that f fits well to a power-law relation of exponent of 1.2 with no inflection point.

V. CONCLUSION

In conclusion we experimentally investigate the depth dependence of plunging force in vertical slow plunging of an intruder in granular medium, and find that the intruder immersion depth dependence of the plunging force has a concave-to-convex transition. Through studying the properties of the inflection point of the concave-convex transition, we have shown that this transition in plunging force is not a result of sidewall support proposed by previous researchers but is a result of filled-in particles on top of the intruder. This argument is verified by plunging a long rod, which results in no inflection in plunging force of depth. In Ref. [18], Hill *et al.* measured both plunging and withdrawing forces, and obtained different depth dependences. Since the exponent of the withdrawing force is greater than that of the plunging force, Hill *et al.* extrapolate their result to deep regime and propose an interesting prediction: although being smaller in shallow regime, the withdrawing force would be greater than the plunging force in a deeper granular bed (about 1 m). However, our finding claims that the sublinear depth dependence of the plunging force cannot be avoided by eliminating the sidewall effect with a larger container, and one cannot extrapolate the power-law dependence obtained in shallow regime to deeper regime. Therefore, the prediction in Ref. [18] should be reconsidered.

ACKNOWLEDGMENTS

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[1] P. G. de Gennes, Rev. Mod. Phys. **71**, s374 (1999).
[2] *The Physics of Granular Media*, edited by H. Hinrichsen and D. E. Wolf (Wiley-VCH, Weinheim, 2004).
[3] R. C. Daniel, A. P. Poloski, and A. E. Saez, Powder Technol. **181**, 237 (2008).
[4] Y. Jiang and M. Liu, Eur. Phys. J. E **22**, 255 (2007).
[5] J. Geng and R. P. Behringer, Phys. Rev. E **71**, 011302 (2005).
[6] E. I. Corwin, H. M. Jaeger, and S. R. Nagel, Nature (London) **435**, 1075 (2005).
[7] T. S. Majmudar and R. P. Behringer, Nature (London) **435**, 1079 (2005).
[8] M. Gerritsen, G. Kreiss, and R. Blumenfeld, Phys. Rev. Lett. **101**, 098001 (2008).
[9] S. Luding, Nature (London) **435**, 159 (2005).
[10] C. Goldenberg and I. Goldhirsch, Nature (London) **435**, 188 (2005).
[11] C. Goldenberg and I. Goldhirsch, Phys. Rev. E **77**, 041303 (2008).
[12] G. I. Tardos, S. McNamara, and I. Talu, Powder Technol. **131**, 23 (2003).
[13] G. I. Tardos, Powder Technol. **92**, 61 (1997).
[14] R. Albert, M. A. Pfeifer, A.-L. Barabási, and P. Schiffer, Phys. Rev. Lett. **82**, 205 (1999).
[15] I. Albert, J. G. Sample, A. J. Morss, S. Rajagopalan, A.-L. Barabasi, and P. Schiffer, Phys. Rev. E **64**, 061303 (2001).
[16] M. B. Stone, D. P. Bernstein, R. Barry, M. D. Pelc, Y. K. Tsui, and P. Schiffer, Nature (London) **427**, 503 (2004).
[17] M. B. Stone, R. Barry, D. P. Bernstein, M. D. Pelc, Y. K. Tsui, and P. Schiffer, Phys. Rev. E **70**, 041301 (2004).
[18] G. Hill, S. Yeung, and S. A. Koehler, Europhys. Lett. **72**, 137 (2005).
[19] Z. Peng, M. Hou, Q. Shi, and K. Lu, Acta Phys. Sin. **56**, 1195 (2007) (in Chinese).
[20] L. Vanel and E. Clement, Eur. Phys. J. B **11**, 525 (1999).
[21] R. Soller and S. A. Koehler, Phys. Rev. E **74**, 021305 (2006).